Quick Tour of Microsoft Excel Solver

Month	Q1	Q2	Q3	Q4	Total
Seasonality	0.9	1.1	0.8	1.2	
Units Sold	3,592	4,390	3,192	4,789	15,962
Sales Revenue	$143,662	$175,587	$127,700	$191,549	$638,498
Cost of Sales	89,789	109,742	79,812	119,718	399,061
Gross Margin	53,873	65,845	47,887	71,831	239,437
Salesforce	8,000	8,000	9,000	9,000	34,000
Advertising	10,000	10,000	10,000	10,000	40,000
Corp Overhead	21,549	26,338	19,155	28,732	95,775
Total Costs	39,549	44,338	38,155	47,732	169,775
Prod. Profit	$14,324	$21,507	$9,732	$24,099	$69,662
Profit Margin	10%	12%	8%	13%	11%

Color Coding

- Target cell
- Changing cells
- Constraints

Product Price	$40.00
Product Cost	$25.00

The following examples show you how to work with the model above to solve for one value or several values to maximize or minimize another value, enter and change constraints, and save a problem model.

Row	Contains	Explanation
3	Fixed values	Seasonality factor: sales are higher in quarters 2 and 4, and lower in quarters 1 and 3.
5	=35*B3*(B11+3000)^0	Forecast for units sold each quarter: row 3 contains the seasonality factor; row 11 contains the cost of advertising.
6	=B5*B18	Sales revenue: forecast for units sold (row 5) times price (cell B18).
7	=B5*B19	Cost of sales: forecast for units sold (row 5) times product cost (cell B19).
8	=B6-B7	Gross margin: sales revenues (row 6) minus cost of sales (row 7).
10	Fixed values	Sales personnel expenses.
11	Fixed values	Advertising budget (about 6.3% of sales).
12	=0.15*B6	Corporate overhead expenses: sales revenues (row 6) times 15%.
13	=SUM(B10:B12)	Total costs: sales personnel expenses (row 10) plus advertising (row 11) plus overhead (row 12).
15	=B8-B13	Product profit: gross margin (row 8) minus total costs (row 13).
16	=B15/B6	Profit margin: profit (row 15) divided by sales revenue (row 6).
18	Fixed values	Product price.
19	Fixed values	Product cost.

This is a typical marketing model that shows sales rising from a base figure (perhaps due to the sales personnel) along with increases in advertising, but with diminishing returns. For example, the first $5,000 of advertising in Q1 yields about 1,092 incremental units sold, but the next $5,000 yields only about 775 units more.

You can use Solver to find out whether the advertising budget is too low, and whether advertising should be allocated differently over time to take advantage of the changing seasonality factor.

Solving for a Value to Maximize Another Value

One way you can use Solver is to determine the maximum value of a cell by changing another cell. The two cells must be related through the formulas on the worksheet. If they are not, changing the value in one cell will not change the value in the other cell.

For example, in the sample worksheet, you want to know how much you need to spend on advertising to generate the maximum profit for the first quarter. You are interested in maximizing profit by changing advertising expenditures.

- On the **Tools** menu, click **Solver**. In the **Set target cell** box, type **b15** or select cell B15 (first-quarter profits) on the worksheet. Select the **Max** option. In the **By changing cells** box, type **b11** or select cell B11 (first-quarter advertising) on the worksheet. Click **Solve**.

You will see messages in the status bar as the problem is set up and Solver starts working. After a moment, you'll see a message that Solver has found a solution. Solver finds that Q1 advertising of $17,093 yields the maximum profit $15,093.

- After you examine the results, select **Restore original values** and click **OK** to discard the results and return cell B11 to its former value.

Resetting the Solver Options

If you want to return the options in the **Solver Parameters** dialog box to their original settings so that you can start a new problem, you can click **Reset All**.

Solving for a Value by Changing Several Values

You can also use Solver to solve for several values at once to maximize or minimize another value. For example, you can solve for the advertising budget for each quarter that will result in the best profits for the entire year. Because the seasonality factor in row 3 enters into the calculation of unit sales in row 5 as a multiplier, it seems logical that you should spend more of your advertising budget in Q4 when the sales response is highest, and less in Q3 when the sales response is lowest. Use Solver to determine the best quarterly allocation.

- On the **Tools** menu, click **Solver**. In the **Set target cell** box, type **f15** or select cell F15 (total profits for the year) on the worksheet. Make sure the **Max** option is selected. In the **By changing cells** box, type **b11:e11** or select cells B11:E11 (the advertising budget for each of the four quarters) on the worksheet. Click **Solve**.
- After you examine the results, click **Restore original values** and click **OK** to discard the results and return all cells to their former values.

You've just asked Solver to solve a moderately complex nonlinear optimization problem; that is, to find values for the four unknowns in cells B11 through E11 that will maximize profits. (This is a nonlinear problem because of the exponentiation that occurs in the formulas in row 5). The results of this unconstrained optimization show that you can increase profits for the year to $79,706 if you spend $89,706 in advertising for the full year.

However, most realistic modeling problems have limiting factors that you will want to apply to certain values. These constraints may be applied to the target cell, the changing cells, or any other value that is related to the formulas in these cells.

Adding a Constraint

So far, the budget recovers the advertising cost and generates additional profit, but you're reaching a point of diminishing returns. Because you can never be sure that your model of sales response to advertising will be valid next year (especially at greatly increased spending levels), it doesn't seem prudent to allow unrestricted spending on advertising.

Suppose you want to maintain your original advertising budget of $40,000. Add the constraint to the problem that limits the sum of advertising during the four quarters to $40,000.

- On the **Tools** menu, click **Solver**, and then click **Add**. The **Add Constraint** dialog box appears. In the **Cell reference** box, type **f11** or select cell F11 (advertising total) on the worksheet. Cell F11 must be less than or equal to $40,000. The relationship in the **Constraint** box is **<=** (less than or equal to) by default, so you don't have to change it. In the box next to the relationship, type **40000**. Click

OK, and then click **Solve**.

- After you examine the results, click **Restore original values** and then click **OK** to discard the results and return the cells to their former values.

The solution found by Solver allocates amounts ranging from $5,117 in Q3 to $15,263 in Q4. Total Profit has increased from $69,662 in the original budget to $71,447, without any increase in the advertising budget.

Changing a Constraint

When you use Microsoft Excel Solver, you can experiment with slightly different parameters to decide the best solution to a problem. For example, you can change a constraint to see whether the results are better or worse than before. In the sample worksheet, try changing the constraint on advertising dollars to $50,000 to see what that does to total profits.

- On the **Tools** menu, click **Solver**. The constraint, **F11<=40000**, should already be selected in the **Subject to the constraints** box. Click **Change**. In the **Constraint** box, change **40000** to **50000**. Click **OK**, and then click **Solve**. Click **Keep solver solution** and then click **OK** to keep the results that are displayed on the worksheet.

Solver finds an optimal solution that yields a total profit of $74,817. That's an improvement of $3,370 over the last figure of $71,447. In most firms, it's not too difficult to justify an incremental investment of $10,000 that yields an additional $3,370 in profit, or a 33.7% return on investment. This solution also results in profits of $4,889 less than the unconstrained result, but you spend $39,706 less to get there.

Saving a Problem Model

When you click **Save** on the **File** menu, the last selections you made in the **Solver Parameters** dialog box are attached to the worksheet and retained when you save the workbook. However, you can define more than one problem for a worksheet by saving them individually using **Save Model** in the **Solver Options** dialog box. Each problem model consists of cells and constraints that you entered in the **Solver Parameters** dialog box.

When you click **Save Model**, the **Save Model** dialog box appears with a default selection, based on the active cell, as the area for saving the model. The suggested range includes a cell for each constraint plus three additional cells. Make sure that this cell range is an empty range on the worksheet.

- On the **Tools** menu, click **Solver**, and then click **Options**. Click **Save Model**. In the **Select model area** box, type **h15:h18** or select cells H15:H18 on the worksheet. Click **OK**.

Note You can also enter a reference to a single cell in the **Select model area** box. Solver will use this reference as the upper-left corner of the range into which it will copy the problem specifications.

To load these problem specifications later, click **Load Model** on the **Solver Options** dialog box, type **h15:h18** in the **Model area** box or select cells H15:H18 on the sample worksheet, and then click **OK**. Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Click **OK** to proceed.

Example 1: Product mix problem with diminishing profit margin.

Your company manufactures TVs, stereos and speakers, using a common parts inventory of power supplies, speaker cones, etc. Parts are in limited supply and you must determine the most profitable mix of products to build. But your profit per unit built decreases with volume because extra price incentives are needed to load the distribution channel.

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			TV set	*Stereo*	*Speaker*
		Number to Build->	100	100	100
Part Name	*Inventory*	*No. Used*			
Chassis	450	200	1	1	0
Picture Tube	250	100	1	0	0
Speaker Cone	800	500	2	2	1
Power Supply	450	200	1	1	0
Electronics	600	400	2	1	1

Diminishing Returns Exponent: 0.9

Profits:

By Product	$4,732	$3,155	$2,208
Total	**$10,095**		

This model provides data for several products using common parts, each with a different profit margin per unit. Parts are limited, so your problem is to determine the number of each product to build from the inventory on hand in order to maximize profits.

Problem Specifications

Target Cell	D18	Goal is to maximize profit.
Changing cells	D9:F9	Units of each product to build.
Constraints	C11:C15<=B11:B15	Number of parts used must be less than or equal to the number of parts in inventory.
	D9:F9>=0	Number to build value must be greater than or equal to 0.

The formulas for profit per product in cells D17:F17 include the factor ^H15 to show that profit per unit diminishes with volume. H15 contains 0.9, which makes the problem nonlinear. If you change H15 to 1.0 to indicate that profit per unit remains constant with volume, and then click **Solve** again, the optimal solution will change. This change also makes the problem linear.

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raints

Example 2: Transportation Problem.

Minimize the costs of shipping goods from production plants to warehouses near metropolitan demand centers, while not exceeding the supply available from each plant and meeting the demand from each metropolitan area.

		Number to ship from plant x to warehouse y (at intersection):				
Plants:	*Total*	*San Fran*	*Denver*	*Chicago*	*Dallas*	*New York*
S. Carolina	5	1	1	1	1	1
Tennessee	5	1	1	1	1	1
Arizona	5	1	1	1	1	1
		---	---	---	---	---
Totals:		3	3	3	3	3
Demands by Whse -->		180	80	200	160	220
Plants:	*Supply*	*Shipping costs from plant x to warehouse y (at intersection):*				
S. Carolina	310	10	8	6	5	4
Tennessee	260	6	5	4	3	6
Arizona	280	3	4	5	5	9
Shipping:	**$83**	$19	$17	$15	$13	$19

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The problem presented in this model involves the shipment of goods from three plants to five regional warehouses. Goods can be shipped from any plant to any warehouse, but it obviously costs more to ship goods over long distances than over short distances. The problem is to determine the amounts to ship from each plant to each warehouse at minimum shipping cost in order to meet the regional demand, while not exceeding the plant supplies.

Problem Specifications

Target cell	B20	Goal is to minimize total shipping cost.
Changing cells	C8:G10	Amount to ship from each plant to each warehouse.
Constraints	B8:B10<=B16:B18	Total shipped must be less than or equal to supply at plant.
	C12:G12>=C14:G14	Totals shipped to warehouses must be greater than or equal to demand at warehouses.
	C8:G10>=0	Number to ship must be greater than or equal to 0.

You can solve this problem faster by selecting the **Assume linear model** check box in the **Solver Options** dialog box before clicking **Solve**. A problem of this type has an optimum solution at which amounts to ship are integers, if all of the supply and demand constraints are integers.

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raints

Example 3: Personnel scheduling for an Amusement Park.

For employees working five consecutive days with two days off, find the schedule that meets demand from attendance levels while minimizing payroll costs.

Sch.	***Days off***	***Employees***		***Sun***	***Mon***	***Tue***	***Wed***	***Thu***	***Fri***	***Sat***
A	*Sunday, Monday*	4		0	0	1	1	1	1	1
B	*Monday, Tuesday*	4		1	0	0	1	1	1	1
C	*Tuesday, Wed.*	4		1	1	0	0	1	1	1
D	*Wed., Thursday*	6		1	1	1	0	0	1	1
E	*Thursday, Friday*	6		1	1	1	1	0	0	1
F	*Friday, Saturday*	4		1	1	1	1	1	0	1
G	*Saturday, Sunday*	4		0	1	1	1	1	1	0
	Schedule Totals:	32		24	24	24	22	20	22	28
	Total Demand:			22	17	13	14	15	18	24

Color Coding

Pay/Employee/Day: $40

Payroll/Week: **$1,280**

The goal for this model is to schedule employees so that you have sufficient staff at the lowest cost. In this example, all employees are paid at the same rate, so by minimizing the number of employees working each day, you also minimize costs. Each employee works five consecutive days, followed by two days off.

Problem Specifications

Target cell	D20	Goal is to minimize payroll cost.
Changing cells	D7:D13	Employees on each schedule.
Constraints	D7:D13>=0	Number of employees must be greater than or equal to 0.
	D7:D13=Integer	Number of employees must be an integer.
	F15:L15>=F17:L17	Employees working each day must be greater than or equal to the demand.
Possible schedules	Rows 7-13	1 means employee on that schedule works that day

In this example, you use an integer constraint so that your solutions do not result in fractional numbers of employees on each schedule. Selecting the **Assume linear model** check box in the **Solver Options** dialog box before you click **Solve** will greatly speed up the solution process.

Target cell

Changing cells

Constraints

Example 4: Working Capital Management.

Determine how to invest excess cash in 1-month, 3-month and 6-month CDs so as to maximize interest income while meeting company cash requirements (plus safety margin).

	Yield	*Term*
1-mo CDs:	1.0%	1
3-mo CDs:	4.0%	3
6-mo CDs:	9.0%	6

Purchase CDs in months:
1, 2, 3, 4, 5 and 6
1 and 4
1

Interest Earned:

Total **$7,700**

Month:	***Month 1***	***Month 2***	***Month 3***	***Month 4***	***Month 5***	***Month 6***	***End***
Init Cash:	$400,000	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400
Matur CDs:		100,000	100,000	110,000	100,000	100,000	120,000
Interest:		1,000	1,000	1,400	1,000	1,000	2,300
1-mo CDs:	100,000	100,000	100,000	100,000	100,000	100,000	
3-mo CDs:	10,000			10,000			
6-mo CDs:	10,000						
Cash Uses:	75,000	(10,000)	(20,000)	80,000	50,000	(15,000)	60,000
End Cash:	$205,000	$216,000	$237,000	$158,400	$109,400	$125,400	$187,700

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If you're a financial officer or a manager, one of your tasks is to manage cash and short-term investments in a way that maximizes interest income, while keeping funds available to meet expenditures. You must trade off the higher interest rates available from longer-term investments against the flexibility provided by keeping funds in short-term investments.

This model calculates ending cash based on initial cash (from the previous month), inflows from maturing certificates of deposit (CDs), outflows for new CDs, and cash needed for company operations for each month.

You have a total of nine decisions to make: the amounts to invest in one-month CDs in months 1 through 6; the amounts to invest in three-month CDs in months 1 and 4; and the amount to invest in six-month CDs in month 1.

Problem Specifications

Target cell	H8	Goal is to maximize interest earned.
Changing cells	B14:G14 B15, E15, B16	Dollars invested in each type of CD.
Constraints	B14:G14>=0 B15:B16>=0 E15>=0	Investment in each type of CD must be greater than or equal to 0.
	B18:H18>=100000	Ending cash must be greater than or equal to $100,000.

The optimal solution determined by Solver earns a total interest income of $16,531 by investing as much as possible in six-month and three-month CDs, and then turns to one-month CDs. This solution satisfies all of the constraints.

Suppose, however, that you want to guarantee that you have enough cash in month 5 for an equipment payment. Add a constraint that the average maturity of the investments held in month 1 should not be more than four months.

The formula in cell B20 computes a total of the amounts invested in month 1 (B14, B15, and B16), weighted by the maturities (1, 3, and 6 months), and then it subtracts from this amount the total investment, weighted by 4. If this quantity is zero or less, the average maturity will not exceed four months. To add this constraint, restore the original values and then click **Solver** on the **Tools** menu. Click **Add**. Type **b20** in the **Cell Reference** box, type **0** in the **Constraint** box, and then click **OK**. To solve the problem, click **Solve**.

To satisfy the four-month maturity constraint, Solver shifts funds from six-month CDs to three-month CDs. The shifted funds now mature in month 4 and, according to the present plan, are reinvested in new three-month CDs. If you need the funds, however, you can keep the cash instead of reinvesting. The $56,896 turning over in month 4 is more than sufficient for the equipment payment in month 5. You've traded about $460 in interest income to gain this flexibility.

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Target cell

Changing cells

Constraints

Example 5: Efficient stock portfolio.

Find the weightings of stocks in an efficient portfolio that maximizes the portfolio rate of return for a given level of risk. This worksheet uses the Sharpe single-index model; you can also use the Markowitz method if you have covariance terms available.

Risk-free rate	6.0%	*Market variance*	3.0%
Market rate	15.0%	*Maximum weight*	100.0%

	Beta	*ResVar*	*Weight*	**Beta*	**Var.*
Stock A	0.80	0.04	20.0%	0.160	0.002
Stock B	1.00	0.20	20.0%	0.200	0.008
Stock C	1.80	0.12	20.0%	0.360	0.005
Stock D	2.20	0.40	20.0%	0.440	0.016
T-bills	0.00	0.00	20.0%	0.000	0.000
Total			100.0%	1.160	0.030
			Return		***Variance***
		Portfolio Totals:	**16.4%**		7.1%

Color Coding

- Target cell
- Changing cells
- Constraints

Maximize Return: A21:A29	**Minimize Risk: D21:D29**
0.1644	0.07077
5	5
1	1
1	1
1	1
1	1
1	1
1	1
1	1

One of the basic principles of investment management is diversification. By holding a portfolio of several stocks, for example, you can earn a rate of return that represents the average of the returns from the individual stocks, while reducing your risk that any one stock will perform poorly.

Using this model, you can use Solver to find the allocation of funds to stocks that minimizes the portfolio risk for a given rate of return, or that maximizes the rate of return for a given level of risk.

This worksheet contains figures for beta (market-related risk) and residual variance for four stocks. In addition, your portfolio includes investments in Treasury bills (T-bills), assumed to have a risk-free rate of return and a variance of zero. Initially equal amounts (20 percent of the portfolio) are invested in each security.

Use Solver to try different allocations of funds to stocks and T-bills to either maximize the portfolio rate of return for a specified level of risk or minimize the risk for a given rate of return. With the initial allocation of 20 percent across the board, the portfolio return is 16.4 percent and the variance is 7.1 percent.

Problem Specifications

Target cell	E18	Goal is to maximize portfolio return.
Changing cells	E10:E14	Weight of each stock.
Constraints	E10:E14>=0	Weights must be greater than or equal to 0.
	E16=1	Weights must equal 1.
	G18<=0.071	Variance must be less than or equal to 0.071.
Beta for each stock	B10:B13	
Variance for each stock	C10:C13	

Cells D21:D29 contain the problem specifications to minimize risk for a required rate of return of 16.4 percent. To load these problem specifications into Solver, click **Solver** on the **Tools** menu, click **Options**, click **Load Model**, select cells D21:D29 on the worksheet, and then click **OK** until the

Solver Parameters dialog box is displayed. Click **Solve**. As you can see, Solver finds portfolio allocations in both cases that surpass the rule of 20 percent across the board.

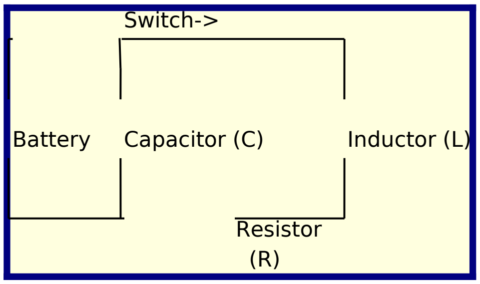
You can earn a higher rate of return (17.1 percent) for the same risk, or you can reduce your risk without giving up any return. These two allocations both represent efficient portfolios.

Cells A21:A29 contain the original problem model. To reload this problem, click **Solver** on the **Tools** menu, click **Options**, click **Load Model**, select cells A21:A29 on the worksheet, and then click **OK**.

Solver displays a message asking if you want to reset the current Solver option settings with the settings for the model you are loading. Click **OK** to proceed.

Example 6: Value of a resistor in an electrical circuit.

Find the value of a resistor in an electrical circuit that will dissipate the charge to 1 percent of its original value within one-twentieth of a second after the switch is closed.



q0 =	9	volts
q[t] =	0.09	volts
t =	0.05	seconds
L =	8	henrys
C =	0	farads
R =	300	ohms
q[t] =	0.25	

Color Coding

Target cell

Changing cells

Constraints

1/(L*C)	1250
(R/(2*L))^2	351.5625
SQRT(B15-B16)	29.973947
COS(T*B17)	0.0720365
-R*T/(2*L)	-0.9375
Q0*EXP(B19)	3.5244506

This model depicts an electrical circuit containing a battery, switch, capacitor, resistor, and inductor. With the switch in the left position, the battery charges the capacitor. When the switch is thrown to the right, the capacitor discharges through the inductor and the resistor, both of which dissipate electrical energy.

Using Kirchhoff's second law, you can formulate and solve a differential equation to determine how the charge on the capacitor varies over time. The formula relates the charge q[t] at time t to the inductance L, resistance R, and capacitance C of the circuit elements.

Use Solver to pick an appropriate value for the resistor R (given values for the inductor L and the capacitor C) that will dissipate the charge to one percent of its initial value within one-twentieth of a second after the time the switch is thrown.

Problem Specifications

Target cell	G15	Goal is to set to value of 0.09.
Changing cell	G12	Resistor.
Constraints	D15:D20	Algebraic solution to Kirchhoff's law.

This problem and solution are appropriate for a narrow range of values; the function represented by the charge on the capacitor over time is actually a damped sine wave.